Model Theory and Mathematical Logic

A conference in honor of Chris Laskowski's 60th birthday

SCHEDULE:

Friday, June 21, 2019:

8:00 - 9:00am: Breakfast.
9:00 - 9:50am: Maryanthe Malliaris - *Model Theory and Ultraproducts.*
10:00 - 10:50am: John Baldwin - *Is $\aleph_1$-Categoricity Absolute for Sentences in $L_{\omega_1,\omega}$.*
11:00 - 11:20am: Coffee Break.
11:20 - 12:10pm: Gabriel Conant - *Generic Stability in Independent Theories.*
12:10 - 2:10pm: Lunch.
2:10 - 3:00pm: David Pierce - *Ratio Then and Now.*
3:10 - 4:00pm: Bradd Hart - *Correspondences and Stability.*
4:10 - 4:30pm: Coffee Break.
4:30 - 5:30pm: Poster Session.

Saturday, June 22, 2019:

8:00 - 9:00am: Breakfast.
9:00 - 9:50am: Caroline Terry - *Speeds of Hereditary Properties and Mutual Algebricity.*
10:00 - 10:50am: John Goodrick - *dp-Minimal Ordered Abelian Groups Revisited.*
11:00 - 11:20am: Coffee Break.
11:20 - 12:10pm: Steffen Lempp (*via Skype*) - *Spectra of Computable Models of Disintegrated Strongly Minimal Theories.*
12:10 - 2:10pm: Lunch.
2:10 - 3:00pm: David Marker - *Model Theory and Machine Learning.*
3:10 - 4:00pm: Alice Medvedev - *Groups in $\mathbb{Q}$ACFA.*
4:10 - 4:30pm: Coffee Break.
4:30 - 5:30pm: Poster Session.
7:00 - 9:00pm: Conference Banquet.

Sunday, June 23, 2019:

8:00 - 9:00am: Breakfast.
9:00 - 9:50am: Saharon Shelah (*via Skype*) - *Simplicity and Universality.*
10:00 - 10:50am: Julia Knight - *Coding and Interpreting Structures.*
11:00 - 11:20am: Coffee Break.
11:20 - 12:10pm: Danul Gunatilleka - *More on Generic Structures.*
12:10 - 2:10pm: Lunch.
2:10 - 3:00pm: Charles Steinhorn - *An Application of o-Minimality in Mathematical Economics.*
3:10 - 4:00pm: Vince Guingona - *Ranks in NIP Theories.*

All talks are held in Kirwan Hall 3206. Coffee breaks are held in Kirwan Hall 3201. The conference banquet is held at Mulligan's Grill and Pub.

Abstracts:

John Baldwin: - *Is $\aleph_1$-Categoricity Absolute for Sentences in $L_{\omega_1,\omega}$*

Abstract: I will survey work Laskowski, Shelah, and I have been working on since at least 2013. A first order theory is categorical in $\aleph_1$ iff it is ω-stable and has no two cardinal models; this characterization is easily seen to be absolute. Already in [She87] Shelah had given an example of an $L_{\omega_1,\omega}(Q)$ sentence what was categorical under MA but not under $2^{\aleph_0} < 2^{\aleph_1}$. He proposed a similar counterexample for $L_{\omega_1,\omega}$, but it was incorrect [She]. The recent work goes in the other direction, trying to find tractable absolute conditions equivalent to $\aleph_1$-categoricity. We reformulate the problem as the study of atomic models of first order theories. In [BLS16], we introduce the notion of pseudo-algebracity and then of a pseudo-minimal set as an analog for strong minimality and prove: If T is a countable first order theory has an atomic model and fewer than $2^{\aleph_1}$ models in $\aleph_1$ then every non-psuedo-algebraic formula is implied by a pseudo-minimal formula. In [LS19], this last conclusion is strengthened to 'pseudo-small' (only countably many types over pcl($\overline{a}$ for finite $\overline{a}$. In [BL19], we expound a generalized Henkin construction which in particular shows every psuedo-minimal theory has a model in the continuum.

References:

[BL19] John T. Baldwin and C. Laskowski. Henkin constructions of models of size continuum. *Bulletin of Symbolic Logic*, 2019. `https://doi.org/10.1017/bsl.2018.2,http://homepage.math.uic.edu/~jbaldwin/pub/henkcontbib`.

[BLS16] John T. Baldwin, C. Laskowski, and S. Shelah. Constructing many atomic models in $\aleph_1$. *Journal of Symbolic Logic*, 81: 1142-1162, 2016.

[LS19] Michael C. Laskowski and Saharon Shelah. On the existence of atomic models. *Archive of Math Logic*, 58:99-118,2019.

[She] Saharon Shelah. Abstract elementary classes near $\aleph_1$ sh88r. revision of Classification of nonelementary classes II, Abstract elementary classes; on the Shelah archive.

[She87] Saharon Shelah. Classification of nonelementary classes II, abstract elementary classes. In John T. Baldwin, editor, *Classification theory (Chicago IL, 1985)*, pages 419-497. Springer, Berlin, 1987. paper 88: Proceedings of the USA-Israel Conference on Classification Theory, Chicago, December 1985; volume 1292 of *Lecture Notes in Mathematics*.

Gabriel Conant: - *Generic Stability in Independent Theories*

Abstract: In NIP theories, the notion of "generic stability" for types (and, more generally, Keisler measures) can be characterized using several equivalent definitions. The goal of this talk is to analyze these various definitions in arbitrary theories. In particular, I will present some combinatorial examples that demonstrate interesting connections between the various forms of generic stability and several classical results from Ramsey theory for finite graphs and hypergraphs. Joint work with K. Gannon.

John Goodrick: - *dp-Minimal Ordered Abelian Groups Revisited*

Abstract: Abstract: Dp-minimality is an analogue of "weight one" for NIP theories. Any weakly minimal stable theory, for instance, is dp-minimal, but also (weakly) o-minimal and C-minimal structures are dp-minimal. There has been considerable progress in the past decade towards understanding the consequences of dp-minimality in various algebraic contexts, such as Pierre Simon's work on dp-minimal ordered structures and Will Johnson's characterization of dp-minimal fields.

In this talk we will consider the case of dp-minimal ordered Abelian groups, and consider questions such as: under what conditions can we obtain piecewise local monotonicity of definable functions? What can we say about unary definable sets, both in densely-ordered and the discretely-ordered case? How might these results generalize to the finite dp-rank case?

Some of the results presented are joint work with Viktor Verbovskiy.

Vince Guingona: - *Ranks in NIP Theories*

Abstract: The notion of dp-rank was developed to measure the complexity of types in NIP theories. We will discuss several notions of rank based off of dp-rank, including op-dimension. We will also examine constructing general frameworks for ranks in model theory.

Danul Gunatilleka: - *More on generic structures*

Abstract: We will explore the behavior of certain generic structures that admit prime models and show that in certain cases that the prime model itself is a generic structure. We will further explore how to extend the AE axiomatization for Shelah-Spencer almost sure theories obtained by Laskowski beyond the realm of Baldwin-Shi hypergraphs.

Bradd Hart: - *Correspondences and Stability*

Abstract: Correspondences are one of the key tools of the modern study of von Neumann algebras. Many arguments involving correspondences have a model theoretic flavour and so it is worth it to work out the theory of correspondences for particular von Neumann algebras. Surprisingly this theory turns out to be stable in the continuous setting and the model theoretic description of the class of models is illuminating. This is joint work with Isaac Goldbring and Thomas Sinclair.

Julia Knight: - *Coding and Interpreting Structures*

Abstract: A *Turing computable embedding* is a Turing operator that maps one class of structures to another so as to preserve isomorphism. The embedding codes the input structure in the output structure. It is interesting when there is an effective decoding. It is also interesting when the decoding is very difficult. Harrison-Trainor, Melnikov, R. Miller, and Montalbán defined very general notions of interpretation, in which the interpreting formulas have no fixed arity. I will discuss some known Turing computable embeddings, including the following.

(1) Marker's embedding of directed graphs in undirected graphs,
(2) Mal'tsev's embedding of fields in 2-step nilpotent groups,
(3) Friedman and Stanley's embedding of graphs in linear orderings.

The first two embeddings come with uniform "effective" interpretations, which give uniform effective decoding. For the third, we do not even have uniform interpretation via $L_{\omega_1\omega}$ formulas.

Steffen Lempp: - *Spectra of Computable Models of Disintegrated Strongly Minimal Theories*

Abstract: The investigation of which countable models of a given first-order theory are computable (i.e., have a copy on the natural numbers with a computable atomic diagram) goes back to the dawn of modern logic. The problem has been most thoroughly investigated in the case of uncountably (but not totally) categorical theories T, since there, by Baldwin and Lachlan, the countable models form an elementary chain of length $\omega + 1$: $M_0 \prec M_1 \prec M_2 \prec \cdots \prec M_\omega$ In 1978, Goncharov showed an example in which some but not all countable models are computable. To simplify presenting his and similar results, we define the spectrum of computable models of T as $SCM(T) = \{a \leq \omega : M_a \text{ is computable}\}$, so Goncharov's result can be phrased as saying that $\{0\}$ is a possible spectrum. Over the next four decades, a fairly small number of other possible spectra have been found, but there are almost no general results restricting which sets can be spectra (other than the obvious upper bound $\Sigma^0_{\omega+3}$). Also, thus far, no differences have been found between uncountably categorical and the more restrictive strongly minimal theories, so we restrict ourselves to the latter case. The first strong negative result on spectra is due to Andrews and A. Medvedev (2014) that for a strongly minimal disintegrated theory T in a finite language, the only possible spectra are $\emptyset, \{0\}$, and $[0, \omega]$, in which case one can effectively (but non-uniformly) reduce to the case of a binary relational language.

In this talk, will present on-going joint work with Andrews vastly extending this result. In particular, we are able to show:

(1) There are exactly seven possible spectra for strongly minimal disintegrated theories in a (possibly infinite) binary relational language.
(2) There are exactly ten possible spectra for strongly minimal disintegrated theories in a relational language of bounded arity in which each relation has Morley rank at most 1.
(3) The only additional possible spectra for strongly minimal disintegrated theories in a relational language of unbounded arity in which each relation has Morley rank at most 1 are of the form $[0, \alpha)$ or $[0, \alpha) \cup \{\omega\}$.
(4) There are at most eighteen possible spectra for strongly minimal disintegrated theories in a ternary relational language.

Maryanthe Malliaris: - *Model Theory and Ultraproducts*

Abstract: The talk will outline some recent interactions of ultrafilters and ultrapowers and model theory with a focus on open problems.

David Marker: - *Model Theory and Machine Learning*

Abstract: In his 1992 Journal of the London Mathematical Society paper Chris Laskowski showed that a class of subsets of a structure uniformly definable by a firstorder formula is a VapnikChervonenkis (VC) class if and only if the formula does

not have the independence property. As VC classes are exactly the classes that are learnable by probably approximately correct learning procedures, this was the first connection between model theory and machine learning. Recently Jim Freitag and Hunter Chase have found several other connections, particularly to on-line learning and query learning. I will survey some of their work.

Alice Medvedev: - *Groups in $\mathbb{Q}ACFA$*

Abstract: $\mathbb{Q}$ACFA is the theory in the language $L := \{0, 1, +, \cdot, \sigma_q : q \in \mathbb{Q}\}$ axiomatized by taking the axioms of ACFA, the theory of difference-closed fields, for each automorphism σ_q. $\mathbb{Q}$ACFA is the model-companion of the theory of fields with a $(\mathbb{Q}, +)$-action.

Any definable group G in $\mathbb{Q}$ACFA is defined in the reduct to $L_q := \{0, 1, +, \cdot, \sigma_q\}$ for some one q. Since the reduct to L_q is ACFA, the classical results on groups definable in ACFA apply; and the L-definable structure of G often turns out to be very much controlled by its L_q-definable struture – at least when the reduct to L_q is has finite rank.

This talk will review ACFA and *finite-rank* groups definable there, and explain how a group definable in $\mathbb{Q}$ACFA is controlled by its *finite-rank* reduct to L_q. Much less is known about infinite-rank definable groups in ACFA, and this talk ignores them entirely.

David Pierce: - *Ratio Then and Now*

Abstract: “Heraclitus holds that the findings of sense-experience are untrustworthy, and he sets up reason [logos, ratio] as the criterion” (Sextus Empiricus)

“It is necessary to know that war is common and right is strife [eris] and all things happen by strife and necessity” (Heraclitus, according to Origen)

(1) Strife has arisen between the historian of mathematics and the mathematician who thinks about the past. One must be both, to understand Euclid’s obscure definition of proportion of numbers. Proportion is sameness of ratio. When this occurs between two pairs of numbers, something should be the same about each pair. In Book VII of the Elements, this can only mean that the Euclidean Algorithm has the same steps when applied to either pair of numbers. From this, despite modern suggestions to the contrary, Euclid has rigorous proofs, not only of what we call Euclid’s Lemma, but also of the commutativity of multiplication.
(2) Apollonius of Perga gives three ways to characterize a conic section: (i) an equation, involving a latus rectum, that we can express in Cartesian form; (ii) the proportion whereby the square on the ordinate varies as the abscissa or product of abscissas; (iii) an equation of a triangle with a parallelogram or trapezoid. The latter equation holds in an affine plane. With the advent of Cartesian methods in 1637, the equation seems to have been forgotten, because it is not readily translated into the lengths (symbolized by single minuscule letters) that Descartes has taught us to work with. With the affine equation, Apollonius can give a proof-without-words of what today we consider a coordinate change, performed with more or less laborious computations.

(3) By interpreting the field where algebra is done in the plane where geometry is done, Descartes does inspire new results. An example still builds on work of an ancient mathematician, Pappus of Alexandria. The model companion of the theory of Pappian affine spaces of unspecified dimension, considered as sets of points with ternary relation of collinearity and quaternary relation of parallelism, is the theory of Pappian affine planes over algebraically closed fields.

Saharon Shelah: - *Simplicity and Universality*

Abstract: Fixing a complete first order theory T, countable for transparency, we had known quite well for which cardinals T has a saturated model. This depends on T of course – mainly of whether it is stable/super-stable. But the older, precursor notion of having a universal notion lead us to more complicated answer, quite partial so far, e.g. the strict order property and even SOP_4 lead to having "few cardinals" (a case of GCH almost holds near the cardinal). Note that e.g. GCH gives a complete uninteresting answer and so is the situation e.g. in the Easton model. It seems that necessarily the answer involves sufficient conditions for non-existence of a universal model (in ZFC) and consistency for additional existence. We conjecture that simplicity of T is crucial in answering this. We shall speak on recent advances: a new criterion covers the simplest non- simple theory, so called T_{feq}. We may also speak complementary results on having a universal model for simple theories.

Charles Steinhorn: - *An application of o-minimality in mathematical economics*

Abstract: This talk deals with preference and utility theory in the context of o-minimal expansions $\mathcal{R}$ of the ordered field of real numbers. We give a description of all preferences that can be defined in such a structure $\mathcal{R}$ and when such preferences admit a utility function.

Caroline Terry: - *Speeds of Hereditary Properties and Mutual Algebricity*

Abstract: A hereditary graph property is a class of finite graphs closed under isomorphism and induced subgraphs. Given a hereditary graph property H, the speed of H is the function which sends an integer n to the number of distinct elements in H with underlying set $\{1, ..., n\}$. Not just any function can occur as the speed of hereditary graph property. Specifically, there are discrete "jumps" in the possible speeds. Study of these jumps began with work of Scheinerman and Zito in the 90's, and culminated in a series of papers from the 2000's by Balogh, Bollobás, and Weinreich, in which essentially all possible speeds of a hereditary graph property were characterized. In contrast to this, many aspects of this problem in the hypergraph setting remained unknown. In this talk we present new hypergraph analogues of many of the jumps from the graph setting, specifically those involving the polynomial, exponential, and factorial speeds. The jumps in the factorial range turned out to have surprising connections to the model theoretic notion of mutual algebricity, which we also discuss. This is joint work with Chris Laskowski.